MATH 115A (CHERNIKOV), SPRING 2016

PROBLEM SET 5

DUE FRIDAY, MAY 6

Problem 1. Do Exercise 1, Section 2.2. Justify each answer.

Problem 2. Do Exercise 1, Section 2.3. Justify each answer.

Problem 3. Let

$$A=\begin{pmatrix}3 & 5 & 1 & -2\\ 2 & -1 & 3 & 4\\ 0 & 2 & 1 & 1\end{pmatrix}, B=\begin{pmatrix}2 & 1 & 0 & 1\\ -3 & 0 & 0 & 1\\ 1 & 3 & 2 & 0\end{pmatrix}, C=\begin{pmatrix}3 & -2 & 1\\ 0 & 1 & 1\\ 1 & 4 & 1\\ -1 & 3 & -1\end{pmatrix}.$$

Compute:

(1) $2A-3B$,
(2) $AC-BC$,
(3) B^tA-A^tB.

Problem 4. Complete the proof of Theorem 2.12.

Problem 5. For each of the following linear transformations T, determine whether T is invertible and justify your answer.

(1) $T:\mathbb{R}^2\to\mathbb{R}^3$ defined by $T(a_1,a_2)=(a_1-2a_2,a_2,3a_1+4a_2)$.
(2) $T:\mathbb{R}^2\to\mathbb{R}^3$ defined by $T(a_1,a_2)=(3a_1-a_2,a_2,4a_1)$.
(3) $T:\mathbb{R}^3\to\mathbb{R}^3$ defined by $T(a_1,a_2,a_3)=(3a_1-2a_3,a_2,3a_1+4a_2)$.
(4) $T:P_3(\mathbb{R})\to P_2(\mathbb{R})$ defined by $T(p(x))=p'(x)$.
(5) $T:M_{2\times 2}(\mathbb{R})\to P_2(\mathbb{R})$ defined by $T\begin{pmatrix}a & b\\ c & d\end{pmatrix}=a+2bx+(c+d)x^2$.
(6) $T:M_{2\times 2}(\mathbb{R})\to M_{2\times 2}(\mathbb{R})$ defined by $T\begin{pmatrix}a & b\\ c & d\end{pmatrix}=\begin{pmatrix}a+b & a\\ c & c+d\end{pmatrix}$.

Problem 6. Let $A,B\in M_{n\times n}(F)$ be given. Show:

(1) If A and B are invertible, then AB is invertible, and $(AB)^{-1}=B^{-1}A^{-1}$.
(2) If A is invertible, then A^t is invertible, and $(A^t)^{-1}=(A^{-1})^t$.
(3) If $AB=I_n$, then A and B are invertible, and $A=B^{-1}$, $B=A^{-1}$.
(4) If $A^2=0$, then A is not invertible.

Problem 7. Let V,W be finite dimensional vector spaces, and let T be an isomorphism. Let V_0 be a subspace of V. Show that $T(V_0)$ is a subspace of W, and that $\dim(V_0)=\dim(T(V_0))$.

Problem 8. Let $T:V\to W$ be a linear transformation, $\dim(V)=n$, $\dim(W)=m$. Let β and γ be ordered bases for V and W, respectively.

Prove that $\operatorname{rank}(T)=\operatorname{rank}(L_A)$ and that $\operatorname{nullity}(T)=\operatorname{nullity}(L_A)$, where $A=[T]_\beta^\gamma$. (Hint: use the previous problem.)